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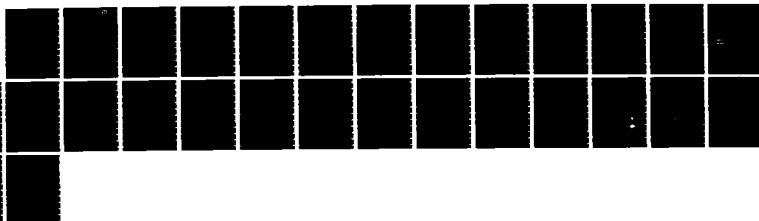
FORTRAM SUBROUTINES FOR ZERO-PHASE DIGITAL FREQUENCY
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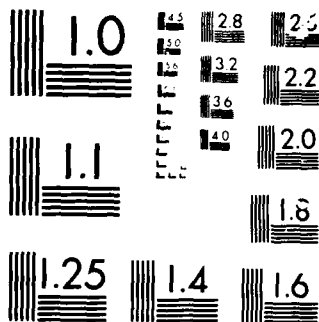
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Special Report 86-4

March 1986



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**US Army Corps
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Cold Regions Research &
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FORTTRAN subroutines for zero-phase digital frequency filters

Donald G. Albert

AD-A168 855

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM	
1. REPORT NUMBER Special Report 86-4		2. GOVT ACCESSION NO. AD-A168855	
4. TITLE (and Subtitle) FORTRAN SUBROUTINES FOR ZERO-PHASE DIGITAL FREQUENCY FILTERS		5. RECIPIENT'S CATALOG NUMBER	
		6. PERFORMING ORG. REPORT NUMBER	
7. AUTHOR(s) Donald G. Albert		8. CONTRACT OR GRANT NUMBER(s)	
9. PERFORMING ORGANIZATION NAME AND ADDRESS U.S. Army Cold Regions Research and Engineering Laboratory Hanover, New Hampshire 03755-1290		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS DA Project 4A762730AT42 Tech. Area B; Tech Effort E, Work Unit 002	
11. CONTROLLING OFFICE NAME AND ADDRESS Office of the Chief of Engineers Washington, D.C. 20314-1000		12. REPORT DATE March 1986	
		13. NUMBER OF PAGES 27	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified	
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution is unlimited.			
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)			
18. SUPPLEMENTARY NOTES			
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Bandpass filters FORTRAN Bandstop filters High pass filters Digital filters Low pass filters			
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report describes and gives user instructions for a series of FORTRAN subroutines that can be used to design and apply zero-phase frequency filters to digitized data. The general properties of these filters are discussed and complete listings are presented.			

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PREFACE

This report was written by Donald G. Albert, Geophysicist, of the Geophysical Sciences Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory. The subroutines were needed for use in DA Project 4A762730AT42, Design, Construction and Operations Technology for Cold Regions; Technical Area B, Combat Development Support; Technical Effort E, Environmental Control Methods (USACRREL); Work Unit 002, Cold Regions Performance of Seismic-Acoustic Sensor Systems.

The author thanks Steven Daly and Gary DeCoff of CRREL for reviewing the manuscript of this report.

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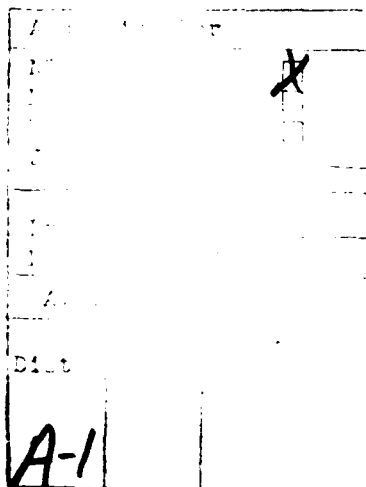
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FORTTRAN SUBROUTINES FOR ZERO-PHASE DIGITAL FREQUENCY FILTERS

Donald G. Albert

INTRODUCTION

This report describes FORTRAN subroutines that can be used to design and apply zero-phase recursive frequency filters to digital data. Four types of filters are included and are described by their transfer function characteristics (Fig. 1): low pass (LP), high pass (HP), bandpass (BP) and bandstop (BS). The subroutines allow the user to remove noise from recorded data or to enhance a selected portion of the signal. The performance of the filters is quite good, and the zero-phase property of the subroutines that apply the filters ensures that no unwanted time shifts in the data will be produced.

Digital filtering is a broad field with a vast amount of literature, and it is not the goal of this report to describe the benefits and shortcomings of various filter types and design procedures, or even to derive all of the properties of the particular filters described here. For readers interested in additional information, Hamming (1983) gives a good

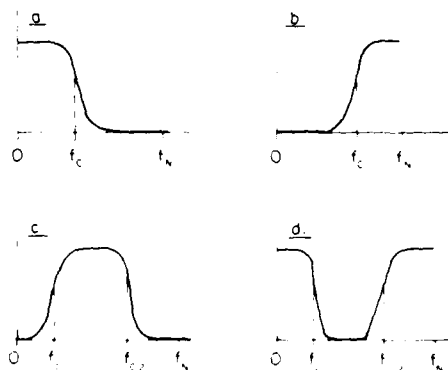


Figure 1. Transfer functions of the four types of filters: a) lowpass, b) highpass, c) bandpass, and d) bandstop

general introduction to the topic of digital filters, while Stearns (1975) provides more mathematical details. Otnes and Enochson (1978) offer some specific examples of digital filtering.

Two types of subroutines are described in this report. The first type designs the filter (calculates the filter coefficients) and is denoted by the suffix DES in the subroutine name. The second type applies the filter (convolves the filter coefficients with the time series data to produce the output series) and is denoted by the suffix FILT. Subroutines LPDES, HPDES and BPDES are reproduced with slight changes from Stearns (1975). Subroutine BSDDES and all of the FILT subroutines were written by the author, using Stearns (1975) as a reference.

The lowpass filter is based on the classical Butterworth analog filter design procedure (Hamming 1983, p 222 ff.). The Butterworth filter is defined by its power gain

$$|H(\omega)|^2 = \frac{1}{1 + \left(\frac{\omega}{\omega_c}\right)^{2N}}$$

where N is the order of the filter and ω_c is the cutoff or design frequency (Fig. 2). $H(\omega)$ is the transfer function of the filter. This filter has maximal smoothness in the pass and the reject band and is tangent at the origin and at infinity. By specifying the amount of ripple allowable in the pass and reject bands, the order and the location of the poles needed to implement the filter can be determined. The bilinear transformation

$$\omega = i \left(\frac{1-z}{1+z} \right)$$

(where i is $\sqrt{-1}$ and z is the new complex variable) is then used to determine the digital filter parameters.

The other filters are also based on the Butterworth filter design procedure, except that a mapping procedure is used to convert the lowpass filter into the highpass, bandpass or bandstop forms (Stearns 1975, Table 12-2) or before the bilinear transformation is used to obtain the digital form. Details are given in Appendix A for the bandstop filter.

To use the digital filters, a design (DES) subroutine is first called with the cutoff frequency (or frequencies), data sample interval and filter order specified. Then a filtering (FILT) subroutine is called to apply the

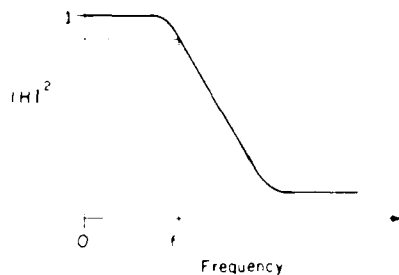


Figure 2. Transfer function of a Butterworth lowpass filter.

filter. This subroutine convolves the filter coefficients with the input data series to produce the filtered output data series. The convolution is then repeated with the data series in reverse order. This second filtering removes any time delays produced by the first filtering process, and the net result is that the filter acts as a zero-phase filter with a transfer function that is the square of the transfer function of the filter specified in the design stage.

The subroutine parameters are listed in the next section. Complete listings are given in Appendix B and are available to users on CRREL's PRIME 9750 computer system. The FILT subroutines require subroutine REVERS which was written by Robinson (1978). This subroutine is also listed in Appendix B.

DESCRIPTION OF SUBROUTINES

Filter design subroutines

Lowpass filter

Calling procedure: CALL LPDES(FC,DT,NS,A,B,C).

Input variables:

FC = cutoff (half-power or -3 dB) frequency in hertz

DT = time interval between samples in seconds

NS = number of filter sections

(2*NS = order of the filter).

Output variables:

A(K),B(K),C(K) = filter coefficients for K = 1,2,...NS; double precision real arrays.

Highpass filter

Calling procedure: CALL HPDES(FC,DT,NS,A,B,C).

Input and output variables: Same as for lowpass filter (above).

Bandpass filter

Calling procedure: CALL BPDES(F1,F2,DT,NS,A,B,C,D,E).

Input variables:

F1 = lower cutoff frequency in hertz

F2 = higher cutoff frequency in hertz.

Output variables:

A(K),...,E(K) = filter coefficients for K=1,2,...NS; double precision
real arrays.

All others same as for lowpass filter (above).

Bandstop filter

Calling procedure: CALL BSDDES(F1,F2,DT,NS,A,B,C,D,E,F,G).

Output variables: A(K),...,G(K) = filter coefficients for K=1,2,...NS;
double precision real arrays.

All others same as for bandpass filter (above).

Filtering subroutines

Lowpass filter

Calling procedure: CALL LPFILT(X,LX,NS,A,B,C).

Input variables:

X = real input data array

LX = integer number of entries in input data array

NS = number of filter sections

A(K),B(K),C(K) = filter coefficients for K=1,2,...NX; double precision
real arrays.

Output variables:

X = real output data array containing the filtered data.

(Note that the input data will be lost.)

Subroutine called: REVERS(X,LX) to reverse the order of a real array
(Robinson 1978).

Highpass, bandpass and bandstop filters

Calling procedures: CALL HPFILT(X,LX,NS,A,B,C)

CALL BPFILT(X,LX,NS,A,B,C,D,E)

CALL BSFILT(X,LX,NS,A,B,C,D,E,F,G).

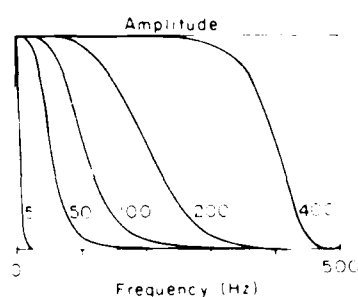
The number of filter coefficients changes for the bandpass and bandstop filter, otherwise all parameters are the same as for the lowpass filter.

USER INSTRUCTIONS

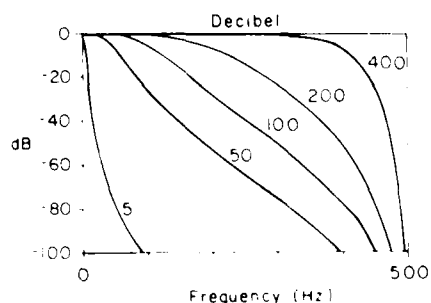
To filter a time series, the desired number of filter sections NS and the cutoff frequency (or frequencies) should be specified in a call to subroutine XXDES, where XX is replaced by the type of filter desired (LP,HP, BP or BS). The time interval DT between samples is fixed by the data time series. Frequencies should be specified in the reciprocal of the units used to specify DT (e.g., hertz if DT is in seconds, cycles/day if DT is in days, etc). The filter coefficients are returned in double precision arrays. These coefficients are then used in subroutine XXFILT to carry out the actual filtering of the data time series.

FILTER PROPERTIES

Figures 3 and 4 illustrate the performance characteristics of some lowpass filters obtained using subroutines LPDES and LPFILT. The performance characteristics of highpass, bandpass and bandstop filters are displayed by the figures in Appendix C. The data sampling interval was 0.001 seconds, and in each case the input data series consisted of 511 zeroes with a single unit-valued spike in the 250th position. Thus, the filter output represents the impulse response of the filter. The Fourier transform of the impulse response of the filter was then calculated and the amplitude spectrum, which is equivalent to the transfer function of the

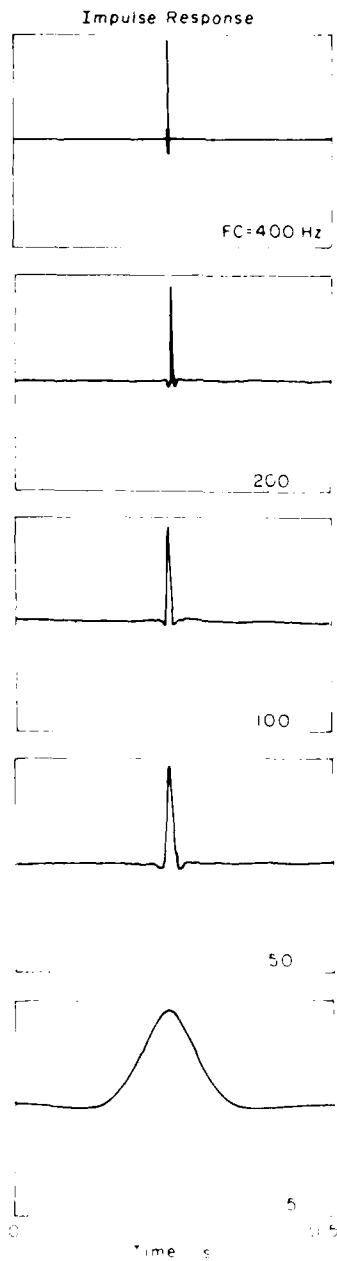


a. Linear display of the transfer function.



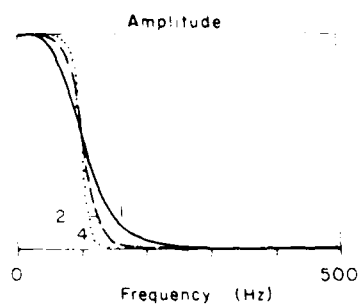
b. Logarithmic display of the transfer function.

Figure 3. Lowpass filter performance characteristics. One stage filters with FC = 400, 200, 100, 50 and 5 Hz.

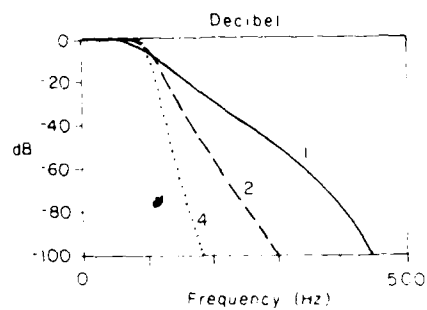


c. Impulse response.

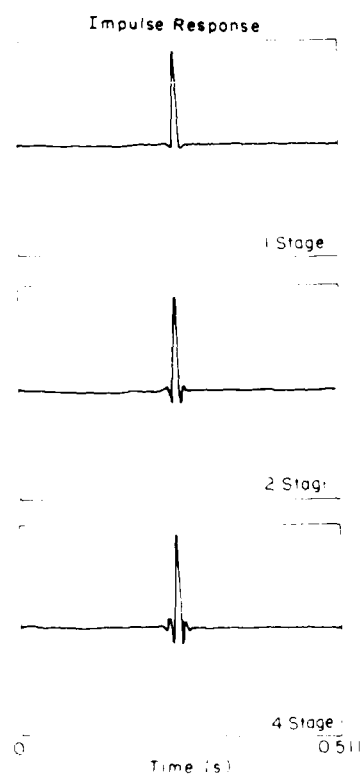
Figure 3 (cont'd). Low-pass filter performance characteristics. One stage filters with FC = 400, 200, 100, 50 and 5 Hz.



a. Linear display of the transfer function.



b. Logarithmic display of the transfer function.



c. Impulse response.

Figure 4. Lowpass filter performance characteristics. One, two and four stage filters with $FC = 100$ Hz.

filter, was found and plotted on a normalized scale. A logarithmic plot of the transfer function is also given in the figures.

The figures show that the impulse response of the filter broadens as the passband of the filter decreases. This property is true for all filters, and the user will sometimes have to compromise between the narrowness of the frequency spectrum and the duration of the signal in the time domain. For example, Figure 3 shows that for a 5-Hz bandwidth, the single spike that was originally 0.001 seconds in duration is now 0.25 seconds wide.

A comparison of the plots in Figure 4 shows that as the number of filter sections increases, the slope of the transfer function between the passband and the stopband steepens. The filter performance improves at the

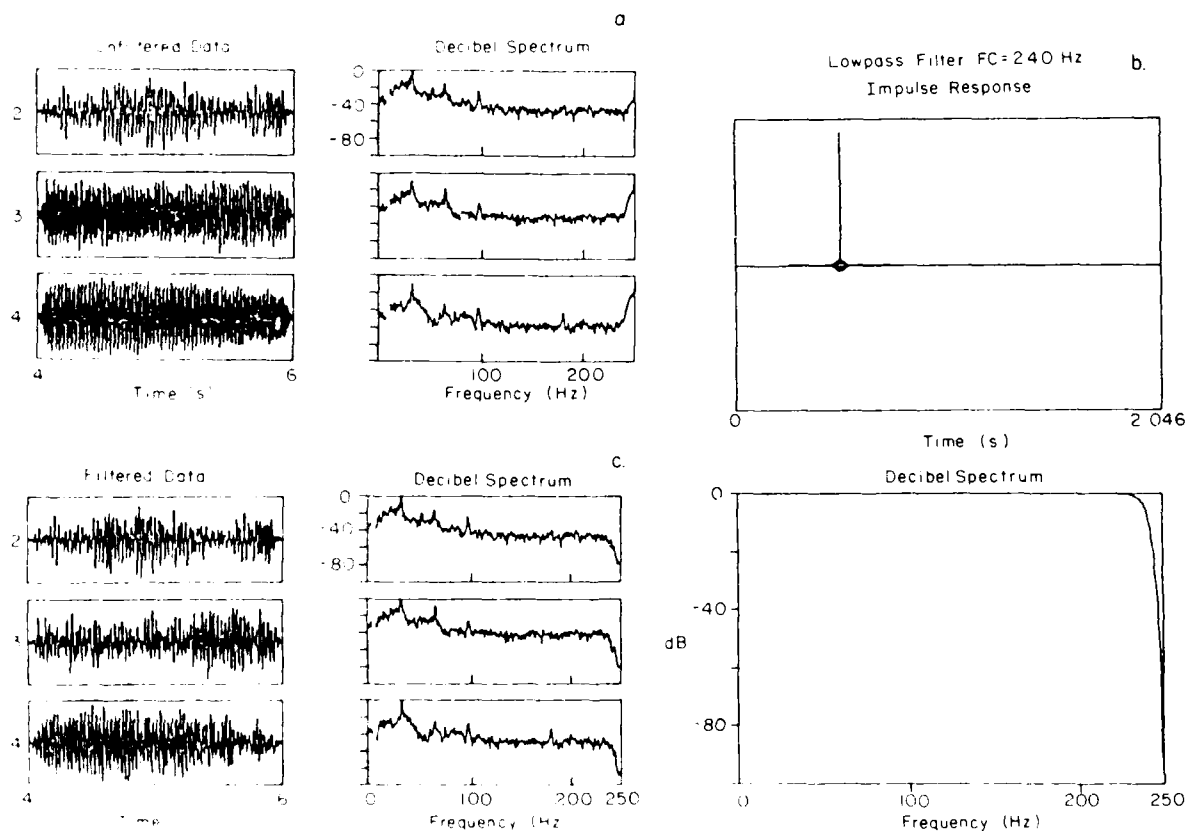


Figure 5. Example of lowpass filtering applied to real data: a) unfiltered data and decibel amplitude spectra, b) lowpass filter impulse response and decibel transfer function, and c) filtered data and decibel amplitude spectra.

expense of increased computational time and larger side lobes in the impulse response.

Figure 5 shows the results of application of a lowpass filter to actual data. In digitizing some seismic data, it was noticed that the digitizing process introduced a strong signal near the Nyquist frequency of 250 Hz (see the increase in decibel spectrum near this frequency in the unfiltered data). This "jitter" was removed by deriving and applying a lowpass filter with a cutoff frequency of 240 Hz, with the results shown.

SUMMARY

FORTRAN subroutines that design and apply zero-phase frequency filters to digital data have been described. User instructions have been included and the general properties of these filters have been pointed out. Complete listings of the subroutines are given in Appendix B.

LITERATURE CITED

- Hamming, R.W. (1983) Digital Filters (2nd ed.). Englewood Cliffs, New Jersey: Prentice-Hall.
- Otnes, R.K. and L. Enochson (1978) Applied Time Series Analysis. Vol. 1, Basic Techniques. New York: Wiley.
- Robinson, E.A. (1978) Multichannel Time Series Analysis with Digital Computer Programs. San Francisco: Holden-Day.
- Stearns, S.D. (1975) Digital Signal Analysis. Rochell Park, New Jersey: Hayden Book Co.

APPENDIX A: DERIVATION OF THE BANDSTOP FILTER

The bandstop filter was derived by applying a frequency transformation that maps a Butterworth lowpass filter transfer function into a bandstop shape. The procedure shown by Stearns (1975, p. 201 ff.) is in two steps:

1. Transform an analog lowpass transfer function $H_{LP}(s)$ into an analog bandstop transfer function $H_{BS}(S)$.
2. Apply the bilinear transformation

$$S = \frac{z-1}{z+1}$$

to convert the analog transfer function to a digital transfer function $\tilde{H}_{BS}(z)$. The Butterworth lowpass analog filter transfer function is

$$H_{LP}(s) = \frac{S_1 S_2}{(S-S_1)(S-S_2)}$$

where $S = i\omega = i2\pi f$. The mapping that converts this transfer function into bandstop form is

$$S \rightarrow \frac{S\omega_c^2}{S^2 + \omega_1\omega_2}$$

(see Fig. A1). Doing the mapping gives

$$H_{BS}(S) = \frac{S^4 + 2\omega_1\omega_2 S^2 + \omega_1^2\omega_2^2}{S^4 - \frac{\omega_c^2(S_1+S_2)}{S_1S_2} S^3 + \left(\frac{\omega_c^4}{S_1S_2} + 2\omega_1\omega_2\right) S^2 - \frac{\omega_c^2(S_1+S_2)}{S_1S_2} \omega_1\omega_2 S + \omega_1^2\omega_2^2}$$

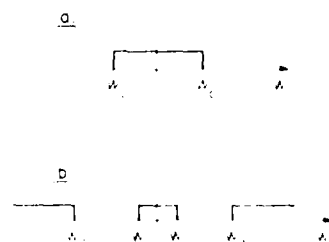


Figure A1. The transformation $S \rightarrow S\omega_c^2/S^2 + \omega_1\omega_2$ converts the lowpass transfer function (a) into a bandstop transfer function (b) (after Stearns 1975, Table 12-1).

If we select ω_1 and ω_2 to be the half-power or -3 dB points, then the poles can be written in exponential form as

$$S_1 = R_e i\theta$$

$$S_2 = R_e^{-i\theta}$$

where

$$R = \omega_c = \omega_2 - \omega_1$$

$$\theta = \pi (2n + N - 1)/2N, \quad n = 1, 2, 3, \dots, 2N$$

and N is the order of the filter. These values are determined by the specification of the Butterworth filter during the design phase. Then

$$S_1 + S_2 = 2 R \cos\theta$$

$$S_1 S_2 = R^2$$

and

$$\frac{\omega_c^2 (S_1 + S_2)}{S_1 S_2} = 2 R \cos\theta$$

$$\frac{\omega_c^4}{S_1 S_2} + 2 \omega_1 \omega_2 = \omega_1^2 + \omega_2^2$$

Substituting these values into the bandstop transfer function gives

$$H_{BS}(S) = \frac{S^4 + 2 \omega_1 \omega_2 S^2 + \omega_1^2 \omega_2^2}{S^4 - 2 R \cos\theta S^3 + (\omega_2^2 + \omega_1^2) S^2 - 2 R \cos\theta \omega_1 \omega_2 S + \omega_1 \omega_2}$$

Applying the bilinear transformation converts this analog transfer function into a digital transfer function:

$$\begin{aligned} \tilde{H}_{BS}(z) &= \frac{\left(\frac{z-1}{z+1}\right)^4 + 2 \omega_1 \omega_2 \left(\frac{z-1}{z+1}\right)^2 + \omega_1^2 \omega_2^2}{\left(\frac{z-1}{z+1}\right)^4 - 2R\cos\theta \left(\frac{z-1}{z+1}\right)^3 + (\omega_2^2 + \omega_1^2) \left(\frac{z-1}{z+1}\right)^2 - 2R\cos\theta \omega_1 \omega_2 \left(\frac{z-1}{z+1}\right) + \omega_1^2 \omega_2^2} \\ &= \frac{(z-1)^4 + 2\omega_1 \omega_2 (z-1)^2 (z+1)^2 + \omega_1^2 \omega_2^2 (z+1)^4}{\{(z-1)^4 - 2R\cos\theta (z-1)^3 (z+1) + (\omega_2^2 + \omega_1^2) (z-1)^2 (z+1)^2 - 2R\cos\theta \omega_1 \omega_2 (z-1) (z+1)^3 \\ &\quad + \omega_1^2 \omega_2^2 (z+1)^4\}} \end{aligned}$$

$$\begin{aligned}
\tilde{H}_{BS}(z) = & \{ [(1+\omega_1\omega_2)^2] (z^4+1) \\
& + [-4(1+\omega_1\omega_2)(1-\omega_1\omega_2)] (z^3+z^2) \\
& + [6 + 6 \omega_1^2 \omega_2^2 - 4 \omega_1 \omega_2] z^2 \} \\
& / \{ [1 + \omega_1^2 \omega_2^2 + 2 \omega_1 \omega_2 - 2(1+\omega_1\omega_2) R \cos \theta] z^4 \\
& + [-4+4 \omega_1^2 \omega_2^2 + 4(1-\omega_1\omega_2) R \cos \theta] z^3 \\
& + [6+6 \omega_1^2 \omega_2^2 - 4 \omega_1 \omega_2] z^2 \\
& + [-4+4 \omega_1^2 \omega_2^2 - 4 (1-\omega_1\omega_2) R \cos \theta] z \\
& + [1+\omega_1^2 \omega_2^2 + 2\omega_1 \omega_2 + 2 (1+\omega_1\omega_2) R \cos \theta] \} .
\end{aligned}$$

A digital bandstop filter with the above transfer function was implemented by the FORTRAN subroutines BSDES and BSFILT.

APPENDIX B: LISTING OF FORTRAN SUBROUTINES

The following subroutines for digital filtering are listed in this appendix: LPDES, HPDES, BPDES, RSDDES, LPFILT, HPFILT, BPFILT, BSFILT, REVERS.

```

C *****
C SUBROUTINE LPDES(FC,DT,NS,A,B,C)
C LOW PASS BUTTERWORTH DIGITAL FILTER DESIGN SBR
C REFERENCE: S.D. STEARNS (1975) DIGITAL SIGNAL ANALYSIS, P. 269.
C INPUTS:
C   FC = CUTOFF FREQUENCY (-3 DB) IN HZ
C   DT = SAMPLING INTERVAL IN SECONDS
C   NS = NUMBER OF FILTER SECTIONS
C OUTPUTS:
C   A(K), B(K), AND C(K), FILTER COEFFICIENTS FOR K=1 TO NS
C   THE DIGITAL FILTER HAS NS SECTIONS IN CASCADE. THE KTH
C   SECTION HAS THE TRANSFER FUNCTION
C
C       
$$H(Z) = \frac{A(K)(Z^2 + 2Z + 1)}{Z^2 + B(K)Z + C(K)}$$

C
C   IF F(M) AND G(M) ARE THE INPUT AND THE OUTPUT OF THE KTH
C   SECTION AT TIME M*DT, THEN
C
C       
$$G(M) = A(K)(F(M) + 2F(M-1) + F(M-2))$$

C       
$$-B(K)G(M-1) - C(K)G(M-2)$$

C
C   REAL*8 A(1),B(1),C(1)
C   PI = 3.1415926536
C   ALPHA = FC*PI*DT
C   WCP = TAN(ALPHA)
C   WCP2 = WCP*WCP
C   DO 10 K = 1, NS
C     CS = COS(FLOAT(2*(K+NS)-1)*PI/FLOAT(4*NS))
C     X = 1./((1.+WCP2-2*WCP*CS))
C     A(K) = WCP2*X
C     B(K) = 2.*(WCP2-1.)*X
C     C(K) = (1.+WCP2+2.*WCP*CS)*X
10  CONTINUE
    RETURN
    END
*
*
C *****
C SUBROUTINE HPDES(FC,DT,NS,A,B,C)
C HIGH PASS BUTTERWORTH DIGITAL FILTER DESIGN SBR
C REFERENCE: S.D. STEARNS (1975) DIGITAL SIGNAL ANALYSIS, P. 270.
C INPUTS:
C   FC = CUTOFF FREQUENCY (-3 DB) IN HZ
C   DT = SAMPLING INTERVAL IN SECONDS
C   NS = NUMBER OF FILTER SECTIONS
C OUTPUTS:
C   A(K), B(K), AND C(K), FILTER COEFFICIENTS FOR K=1 TO NS
C   THE DIGITAL FILTER HAS NS SECTIONS IN CASCADE. THE KTH
C   SECTION HAS THE TRANSFER FUNCTION
C
C       
$$H(Z) = \frac{A(K)(Z^2 - 2Z + 1)}{Z^2 + B(K)Z + C(K)}$$

C
C   IF F(M) AND G(M) ARE THE INPUT AND THE OUTPUT OF THE KTH
C   SECTION AT TIME M*DT, THEN
C
C       
$$G(M) = A(K)(F(M) - 2F(M-1) + F(M-2))$$

C       
$$-B(K)G(M-1) - C(K)G(M-2)$$

C
C   REAL*8 A(1),B(1),C(1)
C   PI = 3.1415926536
C   WCP = SIN(FC*PI*DT)/COS(FC*PI*DT)
C   DO 10 K = 1, NS
C     CS = COS(FLOAT(2*(K+NS)-1)*PI/FLOAT(4*NS))
C     A(K) = 1./((1.+WCP*WCP-2.*WCP*CS))
C     B(K) = 2.*(WCP*WCP-1)*A(K)
C     C(K) = (1.-WCP*WCP+2.*WCP*CS)*A(K)
10  CONTINUE
    RETURN
    END
*
*
C *****

```

```

C *****
C SUBROUTINE BPDES(F1,F2,DT,NS,A,B,C,D,E)
C 3RD ORDER BUTTERWORTH DIGITAL FILTER DESIGN SUB
C REFERENCE: S.O. STEARNS (1975) DIGITAL SIGNAL ANALYSIS, P. 271.
C INPUTS:
C F1,F2 = CUTOFF FREQUENCIES (-3 DB) IN HZ
C DT = SAMPLING INTERVAL IN SECONDS
C NS = NUMBER OF FILTER SECTIONS
C OUTPUTS:
C A(K) THROUGH E(K), FILTER COEFFICIENTS FOR K=1 TO NS
C THE DIGITAL FILTER HAS NS SECTIONS IN CASCADE. THE KTH
C SECTION HAS THE TRANSFER FUNCTION

$$H(Z) = \frac{A(K)*(Z^{**4} - 2*Z^{**2} + 1)}{Z^{**4} + B(K)*Z^{**3} + C(K)*Z^{**2} + D(K)*Z + E(K)}$$

C IF F(M) AND G(M) ARE THE INPUT AND THE OUTPUT OF THE KTH
C SECTION AT TIME M*T, THEN

$$G(M) = A(K)*(F(M) - 2*F(M-2) + F(M-4))$$


$$- B(K)*G(M-1) - C(K)*G(M-2)$$


$$- D(K)*G(M-3) - E(K)*G(M-4)$$

C REAL*8 A(1),B(1),C(1),D(1),E(1)
C PI=3.1415926536
C W1=SIN(F1*PI*DT)/COS(F1*PI*DT)
C W2=SIN(F2*PI*DT)/COS(F2*PI*DT)
C WC=W2-W1
C Q=WC*WC+2.*W1*W2
C S=W1*W1*W2*W2
C DO 10 K=1,NS
C CS=COS(FLOAT(2*(K+NS)-1)*PI/FLOAT(4*NS))
C P=-2.*WC*CS
C R=P*W1*W2
C X=1. + P + Q + R + S
C A(K)=WC*WC/X
C B(K)=( - 4. - 2.*P + 2.*R + 4.*S)/X
C C(K)=(6. - 2.*Q + 6.*S)/X
C D(K)=( - 4. + 2.*P - 2.*R + 4.*S)/X
C E(K)=(1. - P + Q - R + S)/X
10 CONTINUE
C RETURN
C END

```

```

C *****
C SUBROUTINE BSDES(F1,F2,DT,NS,A,B,C,D,E,F,G)
C BANDSTOP BUTTERWORTH DIGITAL FILTER DESIGN SBR
C THIS SBR DERIVED AND DESIGNED BASED ON THE BOOK
C S.J. STEARNS, 1975, DIGITAL SIGNAL ANALYSIS, HAYDEN BOOK CO.
C INPUTS:
C   F1,F2 = CUTOFF FREQUENCIES (-3 DB) IN HZ
C   DT = SAMPLING INTERVAL IN SECONDS
C   NS = NUMBER OF FILTER SECTIONS
C OUTPUTS:
C   A(K) THROUGH E(K), FILTER COEFFICIENTS FOR K=1 TO NS
C THE DIGITAL FILTER HAS NS SECTIONS IN CASCADE. THE KTH SECTION
C HAS THE TRANSFER FUNCTION
C
C 
$$H(Z) = \frac{A(K)*(Z^{**4} + 1) + B(K)*(Z^{**3} + Z) + C(K)*Z^{**2}}{Z^{**4} + D(K)*Z^{**3} + E(K)*Z^{**2} + F(K)*Z + G(K)}$$

C IF F(M) AND G(M) ARE THE INPUT AND OUTPUT OF THE KTH SECTION
C AT TIME M*T, THEN
C
C 
$$G(M) = A(K)*(F(M) + F(M-4))$$

C 
$$+ B(K)*(F(M-1) + F(M-3)) + C(K)*F(M-2)$$

C 
$$- D(K)*G(M-1) - E(K)*G(M-2) - F(K)*G(M-3) - G(K)*G(M-4)$$

C USE SBR BSFILT TO APPLY THE BANDSTOP FILTER
C DON ALBERT 12/21/82
C REAL*8 A(1),B(1),C(1),D(1),E(1),F(1),G(1)
C PI=3.1415926536
C W1=SIN(F1*PI*DT)/COS(F1*PI*DT)
C W2=SIN(F2*PI*DT)/COS(F2*PI*DT)
C Q=W1*W2
C Q2=Q*Q
C QP=1.+Q
C QM=1.-Q
C R=W2-W1
C S=W1*W1 + W2*W2
C DO 10 K=1,NS
C   CS=R*COS(FLOAT(2*(K+NS)-1)*PI/FLOAT(4*NS))
C   X=1./((1.+Q2+S-2.*CS*QP)
C   A(K)=QP*QP*X
C   B(K)=-4.*QP*QM*X
C   C(K)=(6.+6.*Q2-4.*Q)*X
C   D(K)=4.*(-1.*QP*QM+CS*QM) *X
C   E(K)=(6.+6.*Q2-2.*S) *X
C   F(K)=4.*(-1.*QP*QM-CS*QM) *X
C   G(K)=(1.+Q2+S+2.*CS*QP) *X
10 CONTINUE
C RETURN
C END

```

```

C *****
C SUBROUTINE LPFILT(X,LX,NS,A,B,C)
C APPLIES A LOW PASS FILTER TO ARRAY X
C CALLS SBR REVERS
C BASED ON INFO IN STEARNS (1975)
C DON ALBERT 8/26/83
C
  REAL*8 A(1),B(1),C(1)
  REAL*4 X(LX)
  REAL*4 Y(11,3)
C HANDLES FILTERS UP TO NS=10
C APPLIES THE FILTER TWICE, ONCE IN EACH DIRECTION, SO THE PHASE IS ZERO.
  IFLAG=0
5  CONTINUE
  IFLAG=IFLAG+1
  IF(IFLAG.EQ.3) RETURN
  NS1=NS+1
  DO 10 I=1,NS1
    DO 10 J=1,3
      Y(I,J)=0.0
10  CONTINUE
  DO 40 M=1,LX
    Y(1,3)=X(M)
    DO 20 N=1,NS
      Y(N+1,3)=A(N)*(Y(N,3)+2.*Y(N,2)+Y(N,1))
      -B(N)*Y(N+1,2) - C(N)*Y(N+1,1)
20  CONTINUE
  DO 30 N=1,NS1
    DO 30 MM=1,2
      Y(N,MM)=Y(N,MM+1)
30  CONTINUE
  X(M)=Y(NS1,3)
40  CONTINUE
  CALL REVERS(X,LX)
  GOTO 5
  END

```

```

*
*
C *****
C SUBROUTINE HPFILT(X,LX,NS,A,B,C)
C APPLIES A HIGH PASS FILTER TO ARRAY X
C CALLS SBR REVERS
C BASED ON INFO IN STEARNS (1975)
C DON ALBERT 6/30/83
C
  REAL*8 A(1),B(1),C(1)
  REAL*4 X(LX)
  REAL*4 Y(11,3)
C HANDLES FILTERS UP TO NS=10
C APPLIES THE FILTER TWICE, ONCE IN EACH DIRECTION, SO THE PHASE IS ZERO.
  IFLAG=0
5  CONTINUE
  IFLAG=IFLAG+1
  IF(IFLAG.EQ.3) RETURN
  NS1=NS+1
  DO 10 I=1,NS1
    DO 10 J=1,3
      Y(I,J)=0.0
10  CONTINUE
  DO 40 M=1,LX
    Y(1,3)=X(M)
    DO 20 N=1,NS
      Y(N+1,3)=A(N)*(Y(N,3)-2.*Y(N,2)+Y(N,1))
      -B(N)*Y(N+1,2) - C(N)*Y(N+1,1)
20  CONTINUE
  DO 30 N=1,NS1
    DO 30 MM=1,2
      Y(N,MM)=Y(N,MM+1)
30  CONTINUE
  X(M)=Y(NS1,3)
40  CONTINUE
  CALL REVERS(X,LX)
  GOTO 5
  END

```

```

C SUBROUTINE BPFILT(X,LX,NS,A,B,C,D,E)
C APPLIES A BANDPASS FILTER TO ARRAY X
C CALLS SBR REVERS
C BASED ON INFO IN STEARNS (1975)
C CON ALBERT 12/21/82
C
  REAL*8 A(1),B(1),C(1),D(1),E(1)
  REAL*4 X(LX)
  REAL*4 Y(11,5)
C HANDLES FILTERS UP TO NS=10
C APPLIES THE FILTER TWICE, ONCE IN EACH DIRECTION, SO THE PHASE IS ZERO.
  IFLAG=0
  CONTINUE
  IFLAG=IFLAG+1
  IF(IFLAG.EQ.3) RETURN
  NS1=NS+1
  DO 10 I=1,NS1
    DO 10 J=1,4
      Y(I,J)=0.0
  CONTINUE
  DO 40 M=1,LX
    Y(1,5)=X(M)
    DO 20 N=1,NS
      Y(N+1,5)=A(N)*(Y(N,5)-2.*Y(N,3)+Y(N,1))
      -B(N)*Y(N+1,4)-C(N)*Y(N+1,3)
      -D(N)*Y(N+1,2)-E(N)*Y(N+1,1)
  CONTINUE
  DO 30 N=1,NS1
    DO 30 MM=1,4
      Y(N,MM)=Y(N,MM+1)
  CONTINUE
  X(M)=Y(NS1,5)
  CONTINUE
  CALL REVERS(X,LX)
  GOTD 5
  END

```

```

*
*
C *****
C SUBROUTINE BSFILT(X,LX,NS,A,B,C,D,E,F,G)
C APPLIES A BANDSTOP FILTER TO ARRAY X
C CALLS SBR REVERS
C BASED ON INFO IN STEARNS (1975)
C CON ALBERT 12/21/82
C
  REAL*8 A(1),B(1),C(1),D(1),E(1),F(1),G(1)
  REAL*4 X(LX)
  REAL*4 Y(11,5)
C HANDLES FILTERS UP TO NS=10
C APPLIES THE FILTER TWICE, ONCE IN EACH DIRECTION, SO THE PHASE IS ZERO.
  IFLAG=0
  CONTINUE
  IFLAG=IFLAG+1
  IF(IFLAG.EQ.3) RETURN
  NS1=NS+1
  DO 10 I=1,NS1
    DO 10 J=1,5
      Y(I,J)=0.0
  CONTINUE
  DO 40 M=1,LX
    Y(1,5)=X(M)
    DO 20 N=1,NS
      Y(N+1,5)=A(N)*(Y(N,5)+Y(N,1)) + B(N)*(Y(N,4)+Y(N,2))
      +C(N)*Y(N,3)
      -D(N)*Y(N+1,4) -E(N)*Y(N+1,3)
      -F(N)*Y(N+1,2) -G(N)*Y(N+1,1)
  CONTINUE
  DO 30 N=1,NS1
    DO 30 MM=1,4
      Y(N,MM)=Y(N,MM+1)
  CONTINUE
  X(M)=Y(NS1,5)
  CONTINUE
  CALL REVERS(X,LX)
  GOTD 5
  END

```

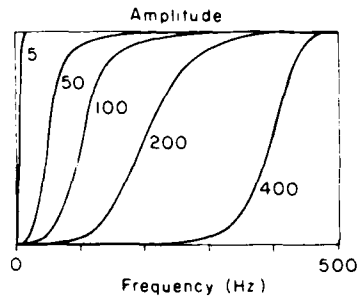
```

*
*
C *****
C SUBROUTINE REVERS(X,LX)
C REVERSES A TIME SERIES
C REFERENCE: ROBINSON (1967), P 28.
C DIMENSION X(LX)
  L=LX/2
  DO 10 I=1,L
    J=LX-I+1
    TEMP=X(I)
    X(I)=X(J)
    X(J)=TEMP
  CONTINUE
  RETURN
  END

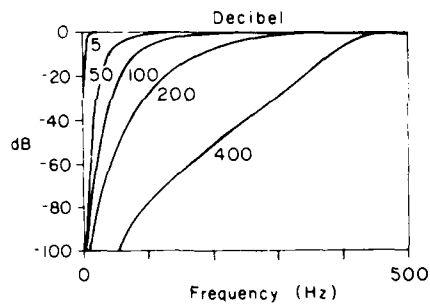
```

APPENDIX C: FILTER PERFORMANCE CHARACTERISTICS

For each figure in this appendix, a filter was applied to an input time series consisting of 511 zeroes with a unit-valued spike at the 250th position to produce the filter's impulse response. The data sampling interval was 0.001 seconds. The Fourier transform of the impulse response was then calculated and used to determine the filter's transfer function, which was plotted on a normalized linear and on a logarithmic scale.



a. Linear display of the transfer function.



b. Logarithmic display of the transfer function.

c. Impulse response (actual data are symmetric about the center).

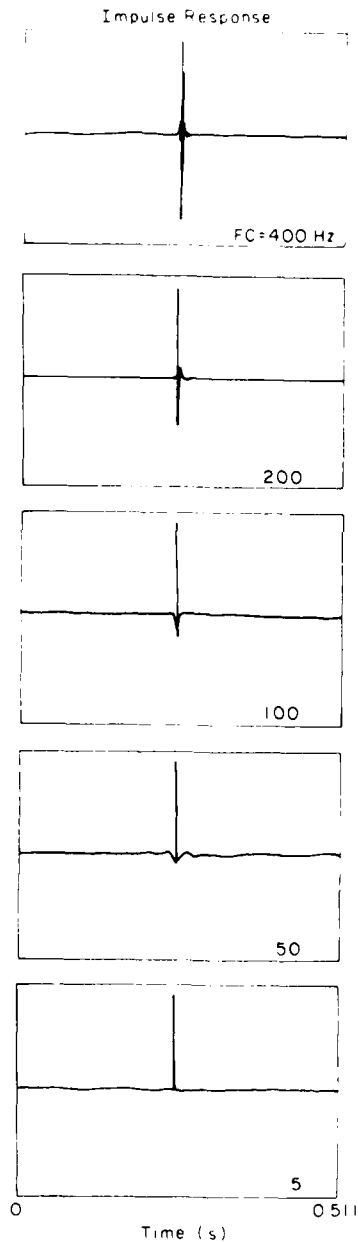
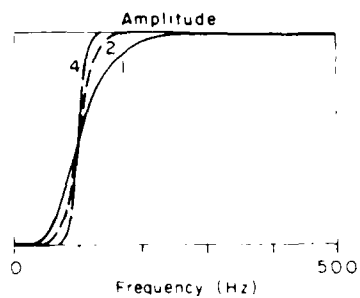
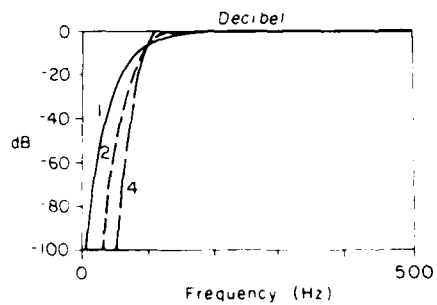


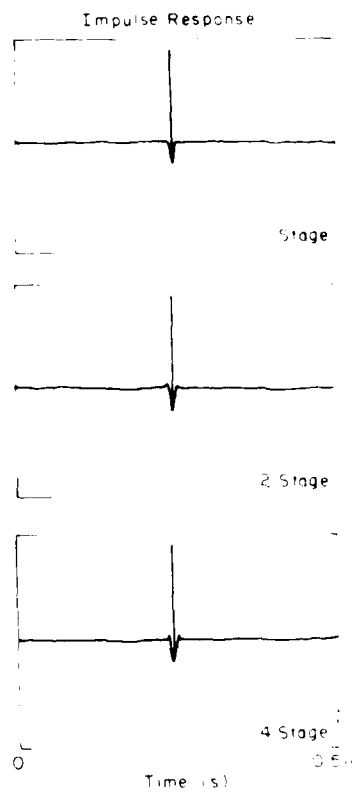
Figure C1. Highpass filter performance characteristics. One stage filters with FC = 400, 200, 100, 50 and 5 Hz.



a. Linear display of the transfer function.

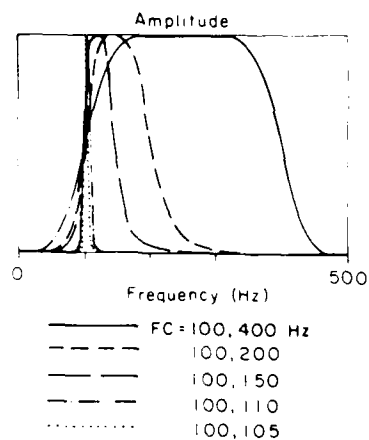


b. Logarithmic display of the transfer function.

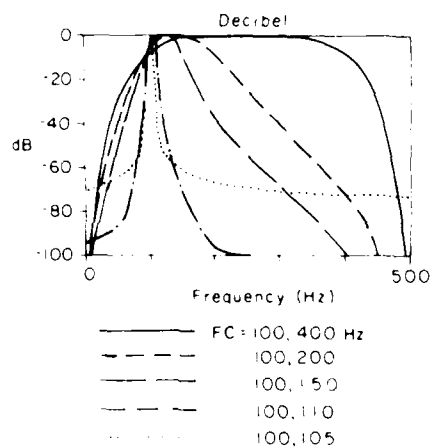


c. Impulse response.

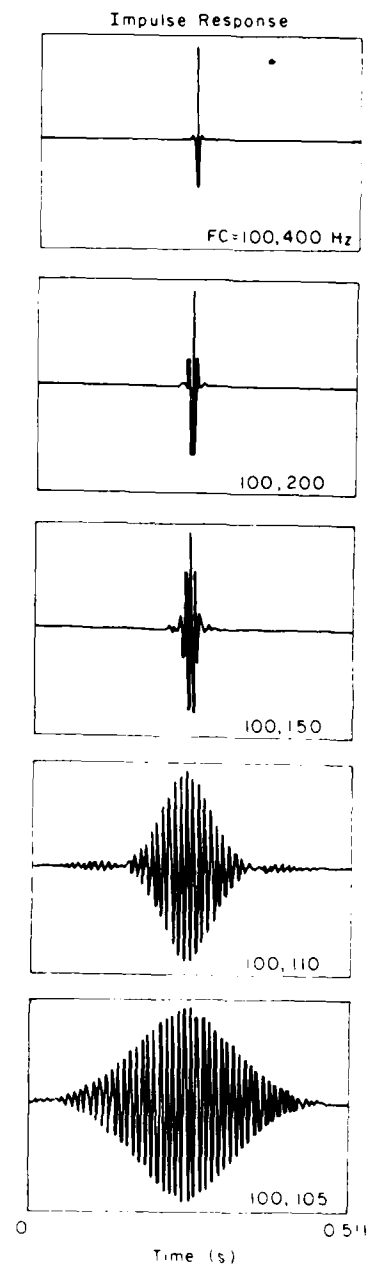
Figure C2. Highpass filter performance characteristics. One, two and four stage filters with $FC = 100$ Hz.



a. Linear display of the transfer function.

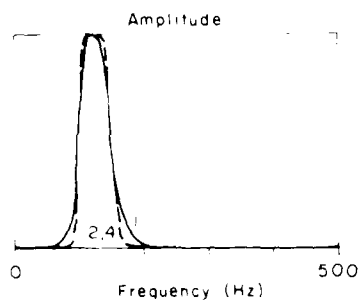


b. Logarithmic display of the transfer function.

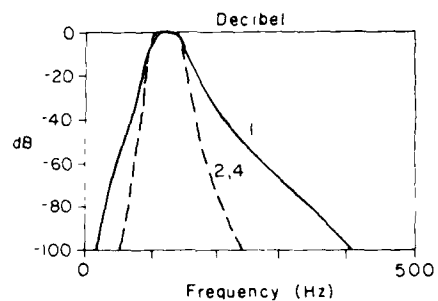


c. Impulse response.

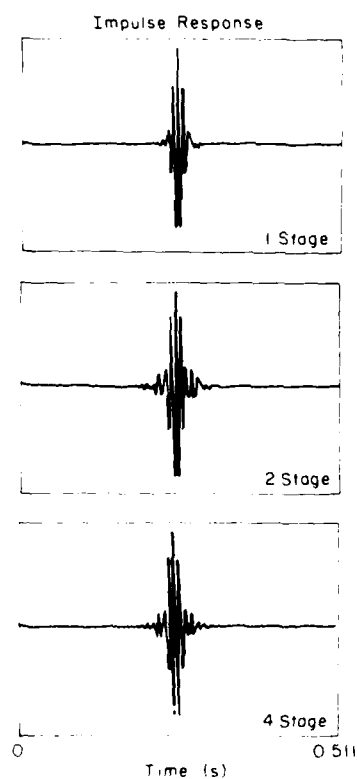
Figure C3. Bandpass filter performance characteristics. One stage filters with lower FC = 100 Hz, upper FC = 400, 200, 150, 110 and 105 Hz.



a. Linear display of the transfer function.

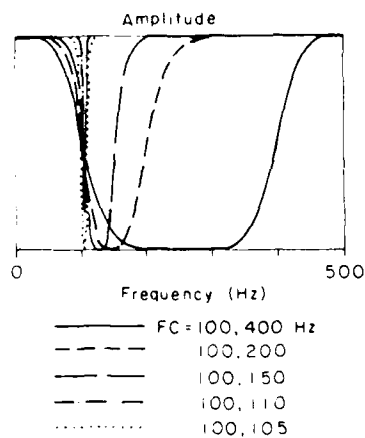


b. Logarithmic display of the transfer function.

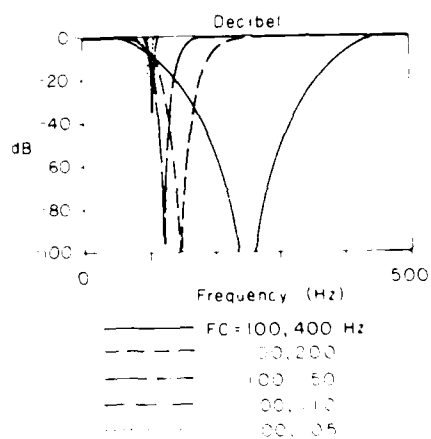


c. Impulse response.

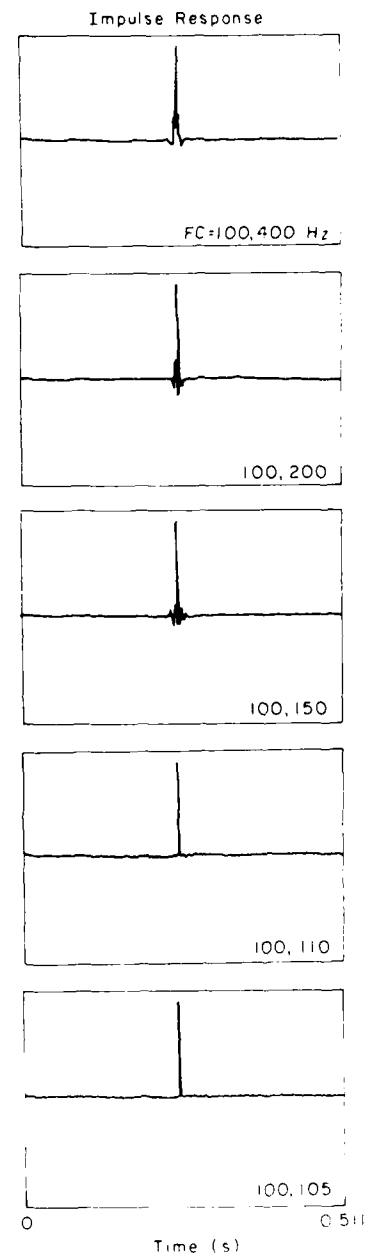
Figure C4. Bandpass filter performance characteristics. One, two and four stage filters with lower FC = 100 Hz, upper FC = 150 Hz.



a. Linear display of the transfer function.

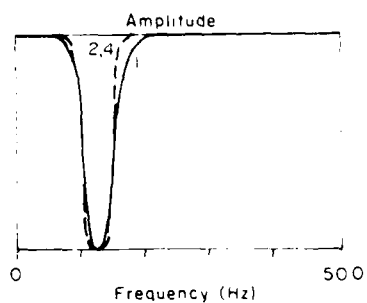


b. Logarithmic display of the transfer function.

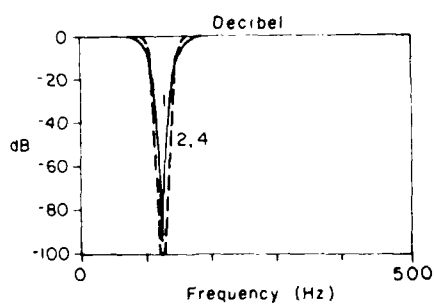


c. Impulse response (actual data are symmetric about the center).

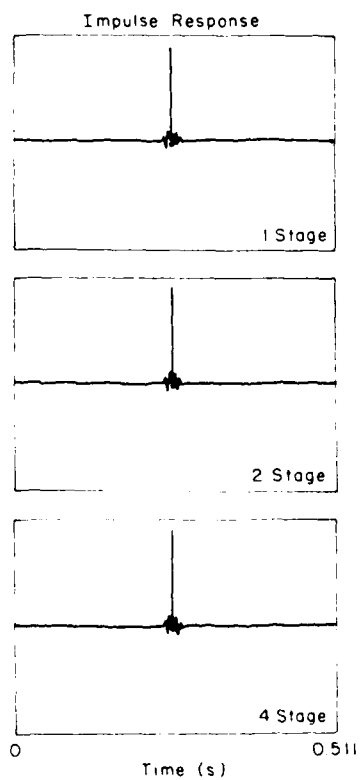
Figure C5. Bandstop filter performance characteristics. One stage filters with lower FC = 100 Hz, upper FC = 400, 200, 150, 110 and 105 Hz.



a. Linear display of the transfer function.



b. Logarithmic display of the transfer function.



c. Impulse response.

Figure C6. Bandpass filter performance characteristics. One, two and four stage filters with lower FC = 100 Hz, upper FC = 150 Hz.

END

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